Analyzing Traffic Signal Delay Models in Tehran Network: A Comprehensive Examination under Varied Traffic Conditions via the Complementary Algorithm

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ABSTRACT

The travel time experienced by users stands as a fundamental determinant of road network service quality at its core. Beyond the inherent delays within road segments, travel time on routes across the network encompasses additional impediments arising from intersections regulated by traffic signals. However, signal delay often remains inadequately addressed within Traffic Assignment models, primarily due to incomplete network information and associated complexities.

This article aims to rectify this oversight by leveraging the complementary Traffic Assignment algorithm to incorporate crucial signal delay functions within the Tehran network context. By doing so, we endeavor to shed light on the nuanced impact of signal delays on travel time dynamics, particularly under both undersaturated and oversaturated traffic conditions.

The application of the complementary Traffic Assignment algorithm allows for a more comprehensive evaluation of travel time, elucidating the intricate interplay between signal delays and overall network performance. Through comparative analysis across varied traffic scenarios, this study seeks to discern the differential effects of signal delays on route efficiency and user experience, thereby offering valuable insights for traffic management and infrastructure optimization efforts.

Keywords: Traffic network, Travel time, Traffic signal delay, complementary algorithm, Tehran network.

1. INTRODUCTION

The process of allocating traffic demand to network arcs is known as Traffic assignment. This process in transportation planning is performed after three stages: Trip generation, Trip distribution, and Modal split. Demand should be allocated to network arcs to reduce drivers' travel time since each driver prefers to cross the road with the lowest travel time to their destination.

The Performance Function is used to calculate delay in network arcs in most Traffic assignment situations. The link between travel time and traffic flow is shown by this function. The BPR (Bureau of Public Roads, 1964) is the most well-known of these functions, having been employed in several Traffic assignment disputes across the globe. The travel time of the arcs in this function is solely determined by the flow inside the arc, and as the flow within the arc grows, so does the travel time of the drivers. The BPR function has the following general form:

$$t = t^0\left(1 + 0{,}15\left(\frac{q}{C}\right)^4\right) \tag{1}$$

In Signalized Intersection, the delay caused by the traffic signal is taken into account in addition to the arc delay. The delay it takes for signal-driven cars to pass through a Signalized Intersection without stopping or decelerating is referred to as this delay. The following three main parts (Roess et al., 2004) split the delay of Signalized Intersection:

Uniform Delay: This delay is estimated based on the assumption that all automobiles reach the intersection simultaneously. The overall delay of each cycle is calculated by calculating the size of the triangle formed by charting the cumulative entrance and departure of vehicles in terms of time.

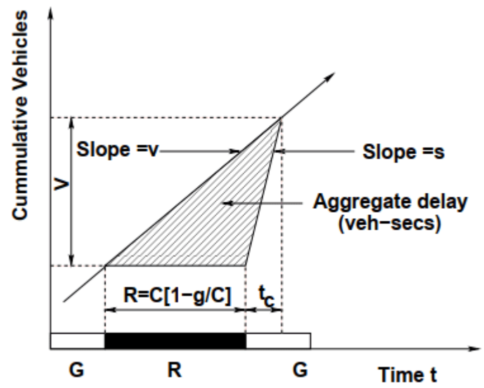

Fig 1. Uniform delay (Roess et al., 2004)

Random Delay*:* This delay modifies a uniform delay by adding or subtracting a number. In reality, it takes into account the impact of cars arriving at the intersection at random. The random entrance of cars into the intersection in some signal cycles causes an Oversaturated wait under undersaturated circumstances, which dissipates in succeeding cycles.

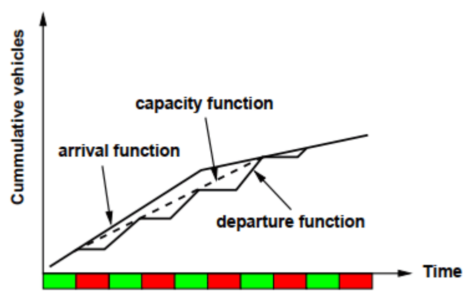

Fig *2*. Random delay (Roess et al., 2004***)***

Overflow Delay: When the capacity of one or more successive phases of the signal is less than the pace at which vehicles enter that phase, an extra delay occurs. Supersaturation queues form at the intersection under oversaturated traffic conditions, and the signal does not vanish in succeeding cycles. There is an Overflow Delay in this situation.

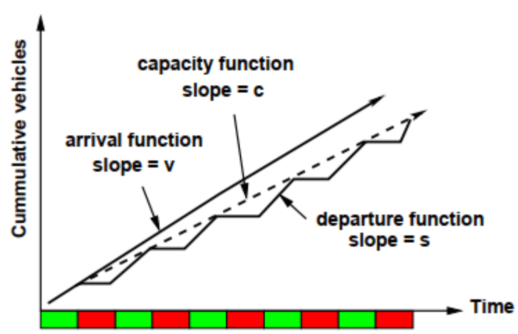

Fig 3. Overflow delay (Roess et al., 2004)

Because the delay experienced at the intersection was signally less than the total travel time of drivers on the entire route in macro assignment models, and because additional information about the traffic signals in a network is required for inserting the signal delay into the model, the traffic signal delay is usually not taken into account. The introduction of the intersection delay function to Traffic assignment models, on the other hand, improves the accuracy of finding network equilibrium flows. The delay function for each entrance street to the intersection can only be calculated using the volume of traffic flow on that street and the green and red times of the signals, and its dependence on the volume of traffic flow on other entrance streets to the intersection, particularly co-phase movements, can be ignored (Ashtiani, & Pourzahidi, 1989; Study of delay time function at intersections, 1996). The total of the arc length travel times and the delay experienced at the intersection of the end of the arc may be used to get the delay function of each arc for this purpose:

$$d_{total} = d_{link} + d_{intersection} \tag{2}$$

As previously stated, using the delay function of Signalized Intersection improves the accuracy of predicting the network's experienced delay. This research aims to see how different kinds of signal delay functions may be used in a static Traffic assignment model. These functions are also used to compare the network's estimated delay.

2. History of traffic signal functions

Various functions pertaining to traffic signals have been examined thus far, as delineated in Table 1.

Table 1. History of Delay Functions (Source: Taken from (Cheng, Du, Sun, & Ji, 2016) cheng et all 2016)

Defect	Advantage	Studies	Research approach	the period	Phase
Random login problem	Ease of calculation	HRB (1928)(Board), 1928), Clayton (1941)(CLAYTON, 1941), Wardrop (1952)(Wardrop, 1952), Newell (1965)(Newell, 1965), and May and Keller (1967)(May Jr & Keller, 1967)	Definitive approach	The1920s–1970s	1
Batch login problem	Probability modeling	Beckmann et al. (1956)(Beckmann, McGuire, & Winsten, 1956) and Newell (1960)(Newell, 1960)	Two-sentence approach		
The problem of overestimation at high saturation degrees	Similar to the real situation	Adams (1936)(Adams, 1936), Webster (1958)(Webster, 1958), Webster and Cobbe (1966)(Webster, 1966), Little (1961)(Little, 1961), Darroch (1964)(Darroch, 1964), Miller (1969)(Miller, 1969), Robertson (1969)(Robertson, 1969), and Ohno (1987))Ohno, 1978(	Poissan's approach		
Need more parameters	Improved accuracy	Miller (1963)(Miller, 1963), Cronje (1983)(Cronje, 1983), and Hutchinson (1972))Hutchinson, 1972(	The main approach		

	Random modeling under saturated and Oversaturated conditions	Catling (1977)(Catling, 1977), Kimber and Hollis (1979)(Kimber & Hollis, 1979), Kimber and Daly (1986), Akcelik (1980, 1981, 1988)(R Akcelik, 1980; Rahmi Akcelik, 1981, 1988), Teply et al. (1984)(Teply, Allingham, Richardson, & Stephenson, 2008), HCM (1985, 1994, 2000)(Manual, 1985, 1994, 2000), Burrow (1989)(Burrow, 1989), Brilon and Wu (1990)(Brilon & Wu, 1990), and Rouphail and Akcelik (1992))Rouphail & Akcelik, 1992(	Time-dependent approach	1970 s-2000s	**2**
	Improved accuracy in time-dependent models	Akcelik (1988)(Rahmi Akcelik, 1988), Fambro and Rouphail (1997)(Fambro & Rouphail, 1997), and Akgungor and Bullen (1999))Akgungor & Bullen, 1999(	Modify the model		
Difficulty estimating more or less reality for specific inputs	Includes progress effects	Reilly et al. (1982)(Reilly, Bolduc, Ken, & Gallagher, 1982), Chang et al. (1987)(Chang, Messer, & Fambro, 1987), Courage et al. (1988)(Courage, Wallace, & Alqasem, 1988), Rouphail (1989)(Rouphail, 1989), Olszewski (1994)(Olszewski, 1994), and Prevedeorus and Koga (1996)(Prevedouros & Koga, 1996)	Progress coefficients		
	Different login templates	Benekohal and El-Zohairy (2001)(Benekohal & El-Zohairy, 2001), Strong et al. (2006)(Strong, Nagui, & Courage, 2006), Ceylan et al. (2007)(Ceylan, Başkan, Ceylan, & Haldenbilen, 2011), Showers (2002)(Showers, 2002), and Kyte et al. (2008))Kyte, Dixon, Nayak, Abdel-Rahim, & Strong, 2008(	Different methods for estimating arrival	2000s onwards	**3**
False predictions	Special traffic conditions	Dion et al. (2004)(Dion, Rakha, & Kang, 2004), Ahmed et al. (2013)(Ahmed, Abu-Lebdeh, & Al-Omari, 2013), Xu et al. (2010)(Xu, Liu, & Tian, 2010), Kikuchi et al. (2004)(Kikuchi, Kii, & Chakroborty, 2004), Zhang and Tong (2008)(Zhang & Tong, 2008), Wang and Benekohal (2007)(Wang & Benekohal, 2007), Yin et al. (2011)(Yin, Zhang, & Wang, 2011), and Zhu et al. (2013))Zhu, Lo, & Lin, 2013(	Delay for queuing and blocking conditions		

There is no specific formula	Control of supersaturation conditions	Gokdag and Hasiloglu (2001)(Gökdağ & Haşiloğlu, 2001), Qiao et al. (2002)(Gökdağ & Haşiloğlu, 2001), Murat and Baskan (2006)(Y. Ş. MURAT, 2006), Murat (2006)(Y. S. Murat & Baskan, 2006), Gokdag et al. (2007)(Gokdag, Hasiloglu, Karsli, Atalay, & Akbas, 2007), Hasiloglu et al. (2014)(Hasiloglu, Gokdag, & Karsli, 2014), and Korkmaz and Akgungor (2017)(Korkmaz & AKGÜNGÖR, 2017)	Artificial intelligence approaches		

Uniform Delay Function (1928):

The devices are expected to enter and depart the intersection at a constant and uniform distance from one another in this example. The *average* delay of each vehicle is calculated by dividing the size of the triangle in Figure 1 (the total delay of all cars impacted by the queue) by the number of vehicles going through the intersection. Uniform delay is another name for this delay (Roess et al., 2004).

$$\mathrm{d} = \frac{\mathrm{c}(1-\lambda)^2}{2(1-\frac{\mathrm{q}}{\mathrm{s}})} = \frac{\mathrm{c}(1-\lambda)^2}{2(1-\lambda\mathrm{x})} \tag{3}$$

Although this *function* properly forecasts the amount of delay encountered at low saturation rates ($\frac{\text{flow}}{\text{capacity}}$), it is unable to reliably anticipate the amount of delay experienced at larger flow rates owing to the influence of cars entering the intersection by mistake (Webster, 1958). Also, under supersaturation situations, this function cannot determine the delay.

Beckmann Delay Function (1955):

Beckmann et al. (1955) proposed that the entrance of cars into the intersection follows a binomial distribution in this function. In this *scenario*, the intersection delay is calculated using the following formula:

$$\mathrm{d} = \frac{\mathrm{c}-\mathrm{g}}{\mathrm{c}(1-\lambda\mathrm{x})}\left[\frac{\mathrm{Q}(0)}{\mathrm{q}} + \frac{\mathrm{c}-\mathrm{g}+1}{2}\right] \tag{4}$$

Webster Delay Function (1958):

Two theoretical terms and one experimental term are used to create this function. The first term of this function represents a uniform *delay*, while the second term represents a random entry delay. The resultant random delay is based on the Poissan distribution of cars approaching the intersection. The third term, which is generally 5 to 15% of the total uniform and *random* delay, is an empirical term that is more in line with what is really occurring. As a result, the total delay in the first two terms may generally be calculated as 0.9 (Webster, 1958):

$$\mathrm{d} = \frac{\mathrm{c}(1-\lambda)^2}{2(1-\lambda\mathrm{x})} + \frac{\mathrm{x}^2}{2\mathrm{q}(1-\mathrm{x})} - 0.65\left(\frac{\mathrm{c}}{\mathrm{q}^2}\right)^{\frac{1}{3}}\mathrm{x}^{(2+5\lambda)} \approx 0.9\left[\frac{\mathrm{c}(1-\lambda)^2}{2(1-\lambda\mathrm{x})} + \frac{\mathrm{x}^2}{2\mathrm{q}(1-\mathrm{x})}\right] \tag{5}$$

One of the model's drawbacks is that the delay tends to be endless as the current approaches the capacity. Under supersaturation *circumstances*, this function, like the previous two, cannot predict the delay.

Miller Delay Function (1963):

For the vehicle's arrival rate at the intersection, all prior functions employed a particular statistical distribution (uniform, poissan, *binomial*, etc.). The traffic signal delay function, according to Miller's study, has a general form for the rate at which vehicles enter the intersection, which can be used to determine the amount of delay at various entry rates depending on the scenario. Miller (1963) estimated the average latency experienced by all vehicles approaching an intersection by computing the experienced delay of automobiles intersecting at the green signal time and cars intersecting at a red signal time separately (Miller, 1963). The following formula is the sum of the two computed values:

$$d = \frac{(1-\lambda)}{2(1-\lambda x)}\left((c-g) + \frac{(2x-1)I}{q(1-x)} + \frac{I+\lambda x-1}{s}\right) \tag{6}$$

The way vehicles enter the intersection is determined by parameter I. This parameter is equal to the variance to mean ratio, which is assumed to be one in the case of random vehicle entry. When I = 1 is taken into account, this formula and the Webster *formula* get comparable results. According to Miller, the first term of this calculation relates to the average uniform delay caused by traffic flow interfering with the traffic signal. The second term additionally shows how much time has passed owing to the lingering backlog at the conclusion of green time. The term is equal to zero at degrees of saturation (x) less than 0.5.

Newell Delay Function (1965):

Newell (1965) offered a model based on Webster's research that examines the general circumstances for vehicles entering an intersection, comparable to Miller's model:

$$d = \frac{c(1-\lambda)^2}{2(1-\frac{q}{s})} + \frac{IH(\mu)x}{2q(1-x)} + \frac{(1-\lambda)I}{2s(1-\lambda x)^2} \tag{7}$$

Cronje (1983) obtained the *functions* μ and $H(\mu)$during the modifications he made to this model as follows:

$$= (1-x)(sg)^{0.5} \tag{8}$$

$$H(\mu) = exp\left[-\mu - \left(\frac{\mu^2}{2}\right)\right] \tag{9}$$

2-2 - Supersaturation mode

May & Keller Delay Function (1967):

As the degree of saturation is low, the functions in the preceding section lead to excellent solutions, but when the current in the arc *approaches* the arc capacity, the delay diagram tends to be infinite, and the answers become irrational (Cheng et al., 2016). Furthermore, under supersaturation circumstances, these functions cannot quantify the delay. In supersaturation circumstances, the amount of time spent in the queue is proportional to the length of time studying, and the length of time studied grows with the length of time studied. May and Keller presented the following discontinuous function for the deterministic study of vehicles in Oversaturated conditions:

$$\mathrm{d} = \frac{\mathrm{c}(1-\lambda)^2}{2(1-\lambda\mathrm{x})} + \frac{\mathrm{T}}{2}(\mathrm{x}-1) \tag{10}$$

This function is equal to the uniform delay function under unsaturated circumstances and does not account for the impact of cars entering *the* junction by mistake. The degree of saturation (x) in the first semester is deemed equal to 1 under supersaturation situations (Roess et al., 2004). It should be noted that under supersaturation circumstances, this function calculates the amount of saturated current delay to be zero at the start of the interval (0T =). T is also a letter that begins with the letter d.

Akcelik Model (Australian delay function) (1980):

As previously stated, the May and Keller model estimates the supersaturation delay for the condensed component under situations where *the* degree of saturation is one, considering the supersaturation mode and the precise entrance of cars into the junction. First, for overcurrent currents, the produced delay tends to be May and Keller diagrams rather than infinite, as Akcelik (1980) adjusted the delay function. Second, a random input component of the device is added to the specified input for the Oversaturated region (Figure 4).

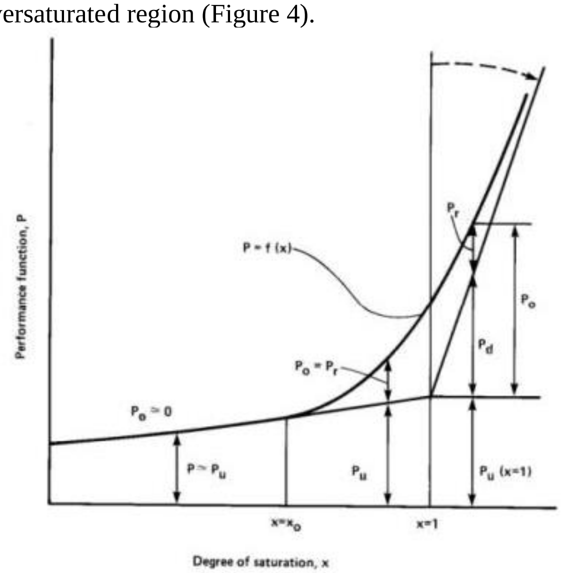

Fig 4. Akcelik, model (1980)

To arrive at the amount of delay suffered, this function first calculates the supersaturation queue as follows, then divides it by the capacity.

$$N_o = \begin{cases} 900 * C * T\left(x - 1 + \sqrt{(x-1)^2 + \frac{12(x - x_0)}{C * T}}\right) & \text{for } x \geq x_0 \\ 0 & \text{for } x < x_0 \end{cases} \quad (11)$$

$$x_0 = 0.67 + \frac{sg}{600} \quad (12)$$

A queue forms at saturation levels greater than x 0. This queue is the consequence of random vehicle entry in portions where the degree of saturation is less than one, but it is higher than a Oversaturated queue in parts where the degree of saturation is more than one.

Finally, here is the signal delay function:

$$d_u = \begin{cases} 0{,}5c(1-\lambda)^2 / \left(1 - \frac{q}{s}\right) & \text{if } x < 1 \\ 0{,}5\,(c - g) & \text{if } x \geq 1 \end{cases} \quad (13)$$

$$d_o = \frac{N_o}{C} \quad (14)$$

$$d = d_u + d_o \quad (15)$$

The Australian delay model is summarized as follows:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + 900T\left[(x-1) + \sqrt{(x-1)^2 + 12\left(\frac{x - x_0}{CT}\right)}\right] \quad (16)$$

$$x_0 = 0.67 + \frac{sg}{600} \quad (17)$$

Canadian delay function model (1984):

The Australian model used to forecast latency at illuminated crossings is quite similar to the Canadian delay model (Teply et al., 1984). *Like* other time-dependent delay models, this one comprises two terms: uniform delay and saturation flow delay. The saturation flow delay calibration coefficients are the sole variation between the Australian and Canadian models. The Canadian delay model is written as follows:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{4x}{CT}}\right] \quad (18)$$

HCM model (Highway Capacity Manual) 1985:

When determining the level of service (LOS) at lit junctions, HCM (HCM, 1985) employs Stopped delay rather than Total delay. In reality, since this model assumes that the Total delay is 1.3 times the Stopped delay, it transforms the Total delay into a Stopped delay by a ratio of 1/3. This model contains a uniform delay term and a saturation current delay term termed Incremental delay, much like prior time-based models; however, unlike the Australian and Canadian models, it only analyzes a defined period of 15 minutes.

The 1985 HCM delay model is as follows:

$$d = 0.38\frac{c(1-\lambda)^2}{(1-\lambda x)} + 173x^2\left[(x-1) + \sqrt{(x-1)^2 + \frac{16x}{C}}\right] \quad (19)$$

Due to the conversion factor and constant analysis duration of 15 minutes, this model's uniform and incremental delay calibration coefficients vary from previous time-dependent delay models. The term $x^{۲}$, which is employed to get better results in *predicting* the delay for saturation situations, is another distinction between the 1985 HCM delay model and the Australian and Canadian delay models. However, under Oversaturated circumstances, the term has been proven to overstate the delay (Akcelik, 1988). This function is likewise not advised for saturation levels greater than 1.2.

Highway Capacity Manual:

Akcelik (1988) suggested *an* alternative to the HCM1985 model. In this model, the delay at saturation degrees below 1 is close to the estimated delay in the HCM1985 model and at saturation degrees above 1 is close to the Australian and Canadian delay models. The alternative model for the 15-minute fixed analysis period is as follows:

$$d = \frac{c(1-\lambda)^2}{2(1-\lambda x)} + 225\left[(x-1) + \sqrt{(x-1)^2 + \frac{32(x-0.5)}{C}}\right] \tag{20}$$

The Stopped delay is *determined* from the following equation when the conversion factor is applied to the alternative model:

$$d = 0.385\frac{c(1-\lambda)^2}{(1-\lambda x)} + 173\left[(x-1) + \sqrt{(x-1)^2 + \frac{32(x-0.5)}{C}}\right] \tag{21}$$

This model has no x^٢ *calibration* term and the second delay term is zero when the saturation degree is less than 0.5.

Highway Capacity Manual:

As previously stated, the HCM1985 model's x^2 term overstated the supersaturation mode delay. Also, at the start of the research period, *this* model was unable to account for the queue; hence it underwent multiple revisions before being presented as follows (HCM, 2010):

$$d = PF * 0.5\frac{c(1-\lambda)^2}{\{1-\lambda[\min(x, 0.1)]\}} + 900T\left[(x-1) + \sqrt{(x-1)^2 + \frac{8kI'x}{C}}\right] + \frac{3600}{qT}\left(t\frac{Q_b + Q_e - Q_{eo}}{2} + \frac{Q_e^2 - Q_{eo}^2}{2c} - \frac{Q_b^2}{2c}\right) \tag{22}$$

Platooning, the influence of upstream signal crossings on the variation of vehicle entrance rates, and the Actuated signal are all elements included in the HCM 2010 model. This model's first semester is about the uniform delay, the second *semester* is about the incremental delay, and the third semester is about the delay induced by the starting queue at the start of the research period. As can be seen, this model does not have a 15-minute analysis constraint, and the duration of the research period has become a factor. This model is also one of the most recent functions for estimating traffic signal junction delay.

3. Problem statement

In order to consider the effect of the amount of delay created by the traffic signal during the journey when solving the problem of Traffic *assignment* for networks with traffic signals, users of the route should, as a function of the travel time of the arcs leading to the traffic signal node, Imported from the traffic signal as the second part of the delay. As a consequence, the bows will have the following travel time function:

Arches leading to traffic signals:

$$t_a(x) = (\alpha_a + \beta_a * x^4) + d(x) \tag{23}$$

Arches without traffic signals:

$$t_a(x) = (\alpha_a + \beta_a * x^4) \tag{24}$$

Using the *Ashtiani* Supplemental Algorithm, we apply the acquired trip time functions to the Tehran network (Aashtiani 1979).

Instead of addressing the Beckman formulation, the Ashtiani Supplemental Algorithm (1979) directly addresses the User-Equilibrium (UE) condition. Using the assumptions, he demonstrated that the EU requirements might be reformulated in terms of flow in the routes as an NCP (Nonlinear Complementarity Problem). The main benefit of this formulation is that, *unlike* Beckman's, each arc's journey time may be described as a function of flow across all arcs in the network. As a result, the Ashtiani algorithm can solve a wider range of Traffic assignment models.

The impact of utilizing the delay function at *junctions* on the effectiveness of Traffic assignment models was explored by Ashtiani and Iravani (1999). The transportation network of Tehran, Iran, was chosen as a case study and modeled in Traffic assignment software (EMME / 2). They utilized the first Webster model expression to create a basic delay function. They looked at the linear correlation (R2) between observed and calculated equilibrium flows, both with and without the delay function. Despite the fact that the model did not account for the incremental delay, their study revealed that employing the delay function raises R2 from 0.69 to 0.75 (when the delay is not taken into account).

The Tehran network has a high traffic volume. Tehran's network has 98,121 knots, 17,790 arcs, and 116,861 origin-destination pairings, with a daily travel demand of 7/721,359,323. This network's related graph is shown in Figure 5. Traffic signals are installed on all nodes with three or more input arcs in this network. The method is coded in C++, and the transit time between pairs of nodes is calculated by combining the delay functions according to formulae 2-1 and 2-2 and applying it to the Tehran network.

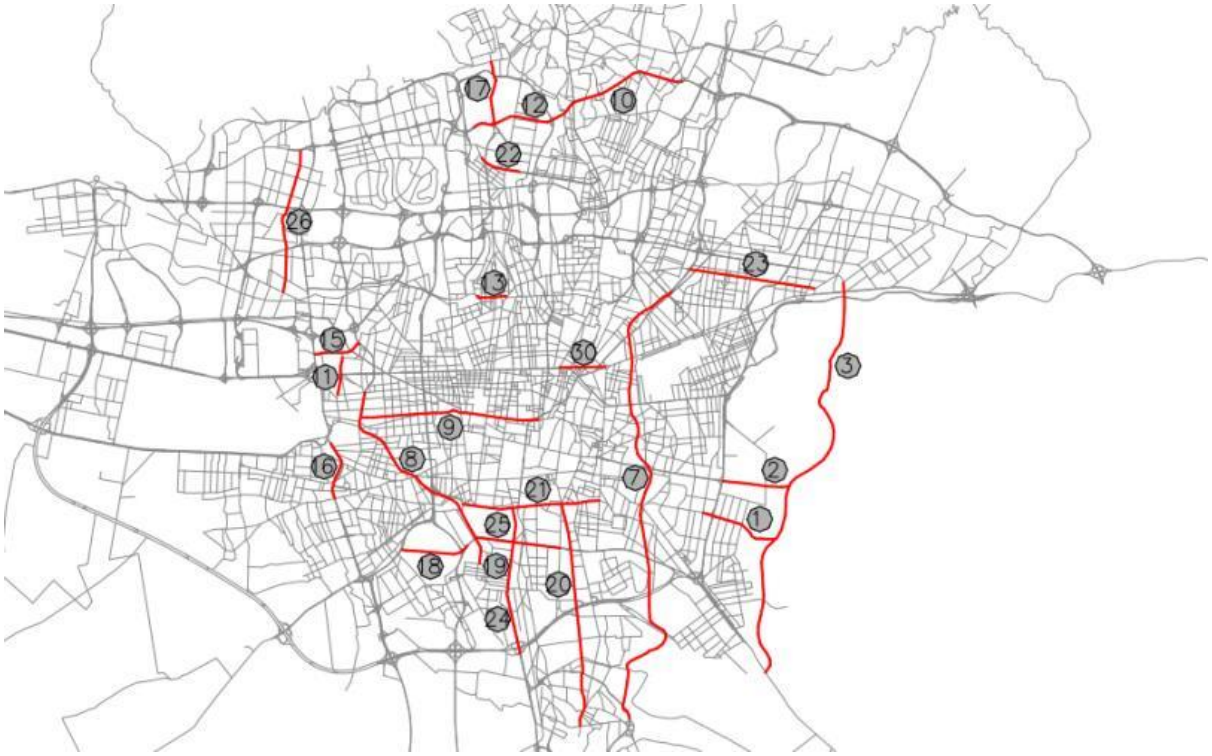

Fig 5. Tehran network

How to apply delay functions:

• Uniform delay model: In unsaturated circumstances, this function is employed.

• Beckman delay model: This function is also employed in unsaturated circumstances. The starting queue value at the start of the research period is assumed to be zero in this study.

• Webster delay model: This model is used in unsaturated conditions, and where the degree of saturation is close to one, this function is extremely desirable.

To apply the aforementioned functions under Oversaturated situations, certain studies, such as comprehensive studies of the city of Tehran, set the denominator of the expressions to a maximum value of 0.01.

• Miller and Noel delay function: These functions are also employed in unsaturated mode. Because the entrance of cars into the junction is deemed random in this research, the value of I is set to one. The Miller function, which displays the remaining queue at the conclusion of the green period, is also deemed equal to zero with degrees of saturation below 0.5 in the second semester.

There is a uniform entry term in all delay functions that may be applied in the Oversaturated state, and the degree of saturation in this *term* must be set to one.

- Australian Delay Model: At saturation degrees below $x_0 = 0{,}67 + \frac{sg}{600}$ the second term (random entry of vehicles) is set to zero.

- HCM 1985 Delay Models and Akcelik Alternatives: As previously stated, these models calculate latency over a 15-minute period. Furthermore, these routines return the stop delay rather than the overall delay. As a result, we multiply the second semester by 4 (study period = 1 hour) and the entire term by 1.3 in this study.

It should be highsignaled that the second semester of the alternative Excel model is assumed to be at saturation degrees of less than 0.5, equal to zero.

- HCM 2010 Delay Model: Isolated traffic signals are used in this study; therefore, the entrance rate of type 3 cars (random entry), the Platon coefficient, and hence the Progression factor (PF) are all one. The influence factor of upstream intersections (I ') is also equal to one owing to the isolation of junctions. Because the signals are pre-timed signals, the incremental delay factor (k) is likewise regarded to be 0.5. It should be noted that the initial queue at the start of the interval is not taken into account in this study; hence the third semester of this function is always zero. The results of this function are calculated using the same assumptions as of the Canadian function. The HCM function calculates the delay for each set of lines independently and then averages the flow weight between them, although the delay functions are employed for the total current of an arc in this research owing to simplicity.

4. Results

The findings of the unsaturated and saturated models are shown in Figures 6 and 7. Figure 6 indicates that in an unsaturated condition, traffic signal delay functions in a reasonably large network such as Tehran are less sensitive to increases in the ratio ($\frac{\text{flow}}{\text{capacity}}$), and their outcomes can only be triggered at lower saturation degrees. However, as shown in Figure 7, the predicted results of the models are well proportionate to each other and are more sensitive to changes in the ratio ($\frac{\text{flow}}{\text{capacity}}$) even at high saturation degrees. Saturation mode models, on the other hand, produce more consistent outcomes than unsaturation mode models.

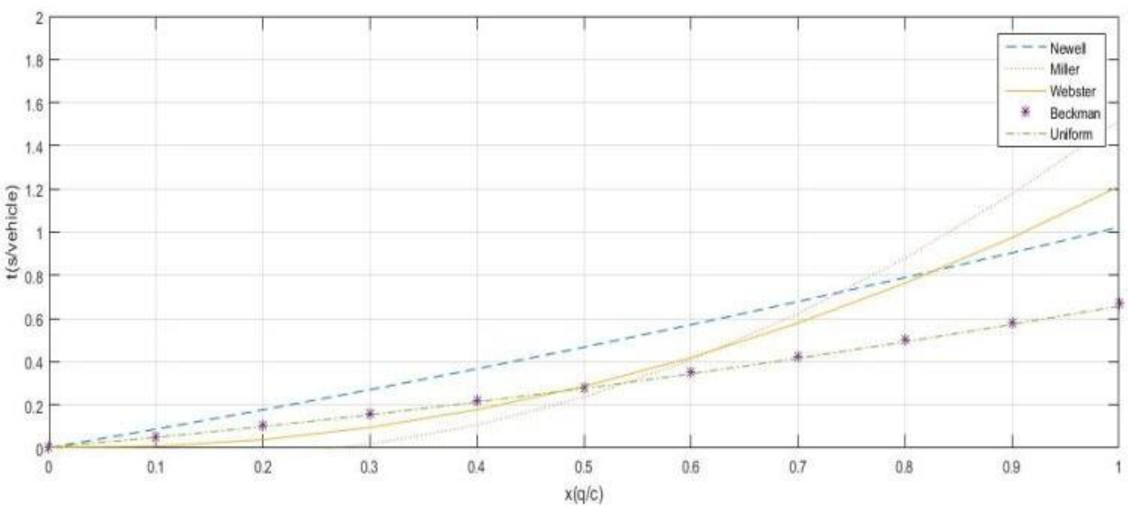

Fig 6. Results of unsaturated models

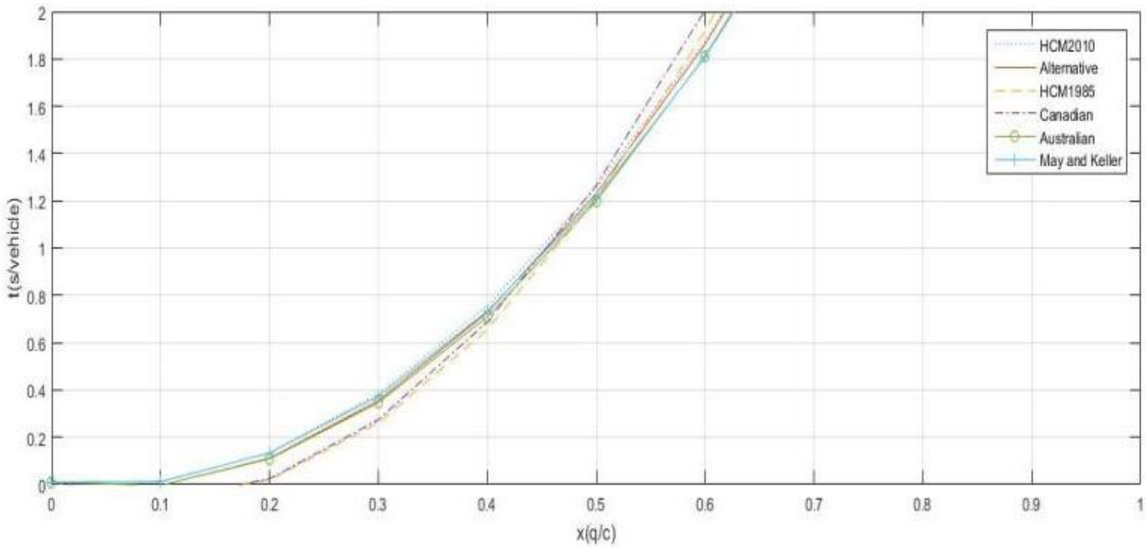

Fig 7. Results of saturation models

We analyzed the findings of vehicle travel time in Tehran based on the use of several unsaturated and saturated state models in Tables 2 and 3. These findings reveal that, when comparing saturated state models to unsaturated state models, comparatively older models, which are often unsaturated state models, have the strongest correlation with saturation state outcomes, but the results of saturation models reflect this. With the assumptions established for their use, these models have produced fairly comparable outcomes.

Table 2. Correlation results between uniform travel time results with other saturation mode models

MODEL	R2
MAY AND KELLER	0.745
AUSTRALIAN	0.743
CANADIAN	0.742
HCM1985	0.716
ALTERNATIVE	0.743
HCM2010	0.744

Table 3. Correlation results between uniform state travel time results with other unsaturated state models

MODEL	R2
DETERMINESTIC	0.827
BECKMAN	0.826
WEBSTER	0.807
MILLER	0.795
NEWELL	0.815

5. Conclusion

Estimating delay at illuminated crossings is a complicated operation that is influenced by a number of factors. Degree of saturation (x) and time of analysis (T) are two of the most crucial variables impacting delay. Despite the fact that the impact of these two factors on delay estimates is well understood and frequently debated, there has been little attempt to effectively incorporate them in delay models.

A comparison of new models to current models revealed that the new models were superior. Also, in a real network, the usage of latency models in conjunction with the arc trip time function has not been examined previously, and in this article, we tried to achieve this important point, and based on the results, the best models were introduced for better implementation in future research.

One proposal for future study is to repeat the procedure on networks larger than the Tehran network, as this is the best model to compare with the results of this paper when considering the delay in illuminated crossings. Another option is to treat traffic signal timing as a new issue rather than a problem to be solved intelligently, then compare the outcomes to the provided models and illustrate the results' compatibility.

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Symptom table:

Symbol	unit	Explanation
d	Second	average delay
d_u	Second	average uniform delay
d_o	Second	average overflow delay
r	Second	Effective red time
g	Second	Effective green time
c	Second	Cycle length time
λ	-	Green time ratio = g / c
s	vps	Saturation flow (departure rate)
q	vps	Link flow (Arrival rate)
C	vph or vps	Link capacity
x	-	Degree of saturation
Q(0)	vehicles	Queue length at the beginning of the period (T = 0)
N_o	vehicles	average overflow queue
I	-	The ratio of variance to average flow per cycle
T	Hours or seconds	Flow measurement time period
x_0	-	The degree of saturation below which the second part of the delay is zero.